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Letter to the Editor

Comments on "Direct fractal measurement of fracture surfaces", *Int. J. Solids and Structures* 36 (1999) 3073–3084, by H. Xie and J. Wang

1. Introduction

Since its outcoming, fractal geometry has provided a new and powerful framework to analyse and explain a wide class of natural phenomena and engineering processes. Researchers involved in this field often have to deal with the in-field estimation of the fractal properties of natural sets. In the paper "Direct fractal measurement of fracture surfaces", Xie et al. claim to overcome the difficulties in the estimation of the fractal dimension of fracture surfaces by means of a new measurement technique called the Projective Covering Method (PCM).

It is worth noting that this method is absolutely not original, but represents only a slight variant of the triangular prism surface area method (also well-known as the patchwork method; Clarke, 1986). In addition, the PCM provides a less efficient estimation of the apparent area which, contrarily to the classical patchwork method, is inherently dependent on the chosen orientation.

The paper by Xie et al. contains several misleading statements on the general fractal theory. In particular, Xie et al. disregard the self-affine nature of fracture surfaces and, consequently, do not correctly recognise the typical transition from the euclidean regime at the largest scales to the fractal behavior shown at the smallest scales. It is also worth mentioning that several methods for the estimation of the fractal dimension of surfaces have been developed, some of which are more effective than the patchwork method when self-affine surfaces are considered.

Last but not least, it is important to stress that, when dealing with the direct measurement of fractal dimension, it is often necessary to consider several orders of magnitude. Otherwise, as probably happened to Xie et al. acquiring only 81×81 points yeilds the evaluated fractal dimension very close to 2.0 (euclidean surface), far from the commonly obtained value $D = 2.2$.

In the following, each of the previous remarks are explained in greater detail.

2. Generalised divider method

The proposed procedure (PCM) to evaluate the fractal dimension of fracture surfaces belongs to the class of the generalised divider methods. All these methods calculate the fractal dimension $D \in [2,3)$ as a function of the scaling properties of the apparent area. It is clear that the term "covering" in the proposed method is itself misleading, the covering methods being a completely different class. Moreover, the proposed formula (namely Eq. (1) in the paper) for the calculation of the apparent area does not overcome the drawbacks of simple profile estimations. This is because the problem of surface anisotropy is not solved. In fact Eq. (1) gives the exact area only if the four considered points lay on a plane.

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Otherwise, if the point heights are not linearly dependent, the calculated area depends on the chosen orientation. This inconsistency can be easily detected by permuting the height values $h_{\rm ak}$, $h_{\rm dk}$, $h_{\rm ck}$ and $h_{\rm bk}$ (we are referring to the same notation proposed in the paper). The magnitude of this variation rapidly increases with the irregularity of the surface. Such behavior does not affect the original method proposed by Clarke, 1986, which was based on a more consistent triangulation scheme for the calculation of the apparent area.

For these reasons, the PCM cannot be addressed as original, nor as an improvement of any existing method.

3. Direct estimation of the fractal dimension of self-affine sets

Fracture surfaces in rocks and concrete, like those observed by Xie et al., hardly ever display self-similar fractal properties, but rather self-affinity. The most famous archetype of these sets is the *Fractional Brownian Surface* (Falconer, 1990; also mentioned by Xie et al.). The main feature of self-affinity is to display different scaling behavior in the height direction with respect to the two orthogonal directions. At small scales, an infinity of details are revealed and the set behaves like a fractal. On the other hand, at large scales, only a few details can be detected, and the surface appears to be an euclidean set. This transition has not been recognised by Xie et al.

In addition, the parameter D_{xy}, which is equal to the mean of the fractal dimensions obtained by means of several orthogonal profiles, clearly contradicts the theorem on the Cartesian product of fractal sets, also cited by Xie et al. (i.e. see Falconer, 1990):

$$\dim(E \times F) \leqslant \dim E + \dim F.$$

It can be useful to remind, here, some of the direct methods able to calculate the fractal dimension of fractal surfaces (Tricot, 1993).

Particularly devoted to self-affine surfaces is the three-dimensional spectral method (Turcotte, 1992). This method considers the mean spectral power (SMP) obtained from the two-dimensional discrete Fourier transform of the surface. The scaling of the SMP gives the means to evaluate the *Hurst exponent*, and then the fractal dimension, of the whole surface.

Unlike the PCM procedure, to provide a correct estimation of the covering dimension, prisms (with topological dimension equal to 3.0) have to be chosen as the basic covering unit. This alternative way is commonly known as *Box Counting Method*. In particular, to obtain a well-defined slope in the bilogarithmic diagram $\log N - \log d$ (where N is the number of boxes and d the box size), the heights of the prisms must be scaled according to d^H (where H is the *Hurst exponent* of the surface).

The experimental data presented in the paper are clearly affected by an excessive rarefaction. Due to the coarse resolution (equal to 0.25 mm, rather than 20 μm like in other similar analyses; Carpinteri et al., 1999), the fractal behavior at small scales is not revealed. This leads to underestimate the fractal dimension, especially when using a divider procedure. Consequently, the calculated fractal dimensions for the considered surfaces are always smaller than 2.1, suggesting wrongly that the surfaces are euclidean rather than fractal. It is worth noting that, regardless of some variations in the fractal dimension of a wide class of fracture surfaces, several authors claim that the value of the *Hurst exponent* $H = 0.8$ (corresponding to a fractal dimension equal to $D = 3-H = 2.2$) is universal (Bouchaud et al., 1990).

The writers have been directly involved in this field in recent years. For references on the subject see Carpinteri et al. (1998) and (1999).

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